

# 8.2

## Perimeter and Area of Composite Figures



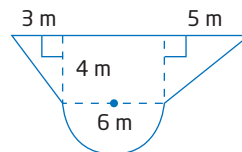
Shapes in everyday life are often made up of several simple shapes. Examples can be seen in logos, architecture, and landscaping. Determining the amount of material needed to construct any of these items may require calculating the perimeter or area of a composite shape.

In this section, you will apply the formulas for the perimeter and area of simple shapes to more complex shapes.

### Investigate

#### How can you apply your knowledge of perimeter and area to a composite figure?

The owners of a restaurant have decided to build an outdoor patio to increase the number of customers that they can serve in the summer. The patio design consists of a rectangle, two right triangles, and a semicircle.



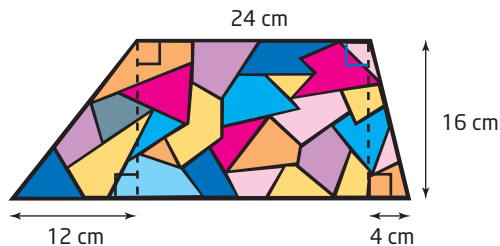
The patio area will be made of interlocking paving stones with different stones along the border. The paving stones cost  $\$52.95/\text{m}^2$ . The border stones are priced according to the length of the border and cost  $\$15.50/\text{m}$ . How much will the materials for the patio cost, including 8% PST and 7% GST? Allow an additional 10% to account for stones that must be cut for the design.

1. Before making any calculations, estimate the cost of the stones for the patio.
2. To calculate the perimeter of the patio, you will need to determine some of the outside measurements.
  - a) Describe how you can calculate the perimeter of the semicircle and the lengths of the two unlabelled sides of the triangles.
  - b) Calculate each of the unknown outside measurements.
  - c) Calculate the total perimeter. Add 10% for waste due to cuts.

3. Now, consider the area of the patio.
  - a) Describe the simple shapes that make up the area.
  - b) Describe how you will calculate the area of each shape.
  - c) Calculate the total area of the patio. Again, add 10% for waste.
4.
  - a) Calculate the cost of the materials for the patio before taxes.
  - b) Calculate the total cost of the materials, including 8% PST and 7% GST.
  - c) Compare this answer to your original estimate. How close were you?
5. **Reflect** Describe an advantage to using simple shapes to calculate the perimeter and area of a composite figure.

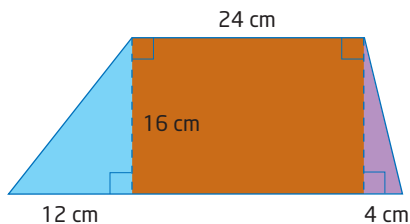
### Example 1 Area and Perimeter of a Composite Figure

- a) Determine the area of the stained-glass panel shown.
- b) Determine the perimeter. Round to the nearest centimetre.



#### Solution

- a) The stained-glass panel can be split into a rectangle and two right triangles.



To find the total area of the panel, add the area of the rectangle and the areas of the two right triangles. Use the formulas for the areas of these shapes.

$$\begin{aligned}
 \text{Call the area of the rectangle } A_{\text{rectangle}}. \\
 A_{\text{rectangle}} &= lw \\
 &= (24)(16) \\
 &= 384
 \end{aligned}$$

Understand the Problem

Choose a Strategy

Carry Out the Strategy

Call the area of the triangle on the left  $A_{\text{triangle 1}}$ .

$$\begin{aligned} A_{\text{triangle 1}} &= \frac{1}{2}bh \\ &= \frac{1}{2}(12)(16) \\ &= 96 \end{aligned}$$

Call the area of the triangle on the right  $A_{\text{triangle 2}}$ .

$$\begin{aligned} A_{\text{triangle 2}} &= \frac{1}{2}bh \\ &= \frac{1}{2}(4)(16) \\ &= 32 \end{aligned}$$

Call the total area  $A_{\text{total}}$ .

$$\begin{aligned} A_{\text{total}} &= A_{\text{rectangle}} + A_{\text{triangle 1}} + A_{\text{triangle 2}} \\ &= 384 + 96 + 32 \\ &= 512 \end{aligned}$$

The total area of the stained-glass panel is  $512 \text{ cm}^2$ .

This stained-glass panel is in the shape of a trapezoid.

Another way to calculate the area of this figure is to use the

formula for a trapezoid,  $A = \frac{1}{2}h(a + b)$  or  $A = \frac{h(a + b)}{2}$ .

$$\begin{aligned} A &= \frac{1}{2}h(a + b) \\ &= \frac{1}{2}(16)(24 + 40) \\ &= 8(64) \\ &= 512 \end{aligned}$$

The longer parallel side is  $(12 + 24 + 4)$  or  $40 \text{ cm}$ .

This gives the same answer as the other method.

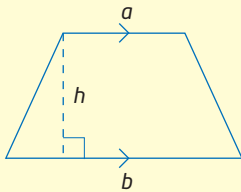
Which method was easier?

The area of the stained-glass panel is  $512 \text{ cm}^2$ .

### Reflect

Is there another way to solve the problem?

The parallel sides of the trapezoid are the  $a$  and  $b$  in the formula. The distance between the parallel sides is the height,  $h$ .

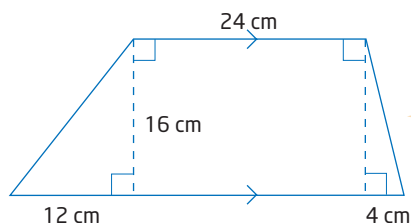


### Understand the Problem

### Choose a Strategy

- b) The perimeter of the stained-glass panel includes two unknown side lengths.

When the figure is split into a rectangle and two right triangles, each unknown side is in a triangle. Apply the Pythagorean theorem to determine the lengths of the two unknown sides in the perimeter.



In both triangles, the unknown side is the hypotenuse.

First, find the length of the unknown side on the left. Call it  $c$ .

$$c^2 = 12^2 + 16^2$$

$$c^2 = 144 + 256$$

$$c^2 = 400$$

$$c = \sqrt{400}$$

$$c = 20$$

Next, find the length of the unknown side on the right. Call it  $d$ .

$$d^2 = 4^2 + 16^2$$

$$d^2 = 16 + 256$$

$$d^2 = 272$$

$$d = \sqrt{272}$$

$$d \doteq 16$$

Now, find the perimeter by adding the outside measurements.

$$P = 24 + 16 + 40 + 20$$

$$= 100$$

The perimeter of the stained-glass panel is approximately 100 cm.

The two unknown sides of the trapezoid must each be longer than 16 cm. This means that the total perimeter must be longer than  $(24 + 16 + 40 + 16)$  or 96 cm. A perimeter of 100 cm seems reasonable for this stained-glass panel.

## Example 2 Area of a Composite Figure, by Subtraction, and Perimeter

A hotel is remodelling its outdoor entrance area. The new design includes a tile walkway leading to a semicircular fountain.

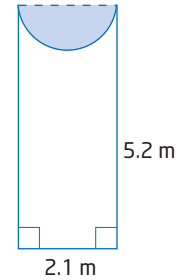
- Describe the steps you would use to find the area of the walkway.
- Calculate the area of the walkway. Round to the nearest tenth of a square metre.
- The walkway will have a border in a different colour of tile. Calculate the perimeter of the walkway. Round to the nearest tenth of a metre.



## Solution

- a) The walkway is a rectangle with a semicircle cut out of it.

Determine the area of the rectangle minus the area of the semicircle.



$$\begin{aligned} \text{b) } A_{\text{rectangle}} &= lw \\ &= (5.2)(2.1) \\ &= 10.92 \end{aligned}$$

Estimate:  $5 \times 2 = 10$

The radius,  $r$ , is half the diameter.

So,  $r = 2.1 \div 2$  or 1.05.

$$A_{\text{semicircle}} = \frac{1}{2}\pi r^2$$

A semicircle is half a circle. So, the area of a semicircle is  $\frac{1}{2}$  the area of a circle.

$$= \frac{1}{2}\pi(1.05)^2$$

Estimate:  $0.5 \times 3 \times 1 \times 1 = 1.5$

$$\doteq 1.73$$

1/2\*pi\*1.05^2  
1.73180295

$$\begin{aligned} A_{\text{walkway}} &= A_{\text{rectangle}} - A_{\text{semicircle}} \\ &= 10.92 - 1.73 \\ &= 9.19 \end{aligned}$$

The area of the walkway is approximately  $9.2 \text{ m}^2$ .

- c) The perimeter of the walkway consists of the three sides of the rectangular section and the semicircular arc.

First, find the length of the semicircular arc.

$$\begin{aligned} L &= \frac{1}{2}(\pi d) \\ &= \frac{1}{2}\pi(2.1) \\ &\doteq 3.3 \end{aligned}$$

The formula for the circumference of a circle is  $C = \pi d$ . So, the length of a semicircular arc is half the circumference.

Now, add the distances around the outside of the walkway.

$$\begin{aligned} P_{\text{walkway}} &= L + \text{three sides of rectangle} \\ &= 3.3 + (5.2 + 2.1 + 5.2) \\ &= 15.8 \end{aligned}$$

The perimeter of the walkway is about 15.8 m.

## Key Concepts

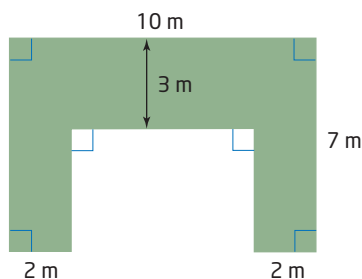
- A composite figure is made up of more than one simple shape.
- To determine the total area of a composite figure, add and/or subtract areas.
- To determine the perimeter of a composite figure, add the distances around the outside of the figure.

## Communicate Your Understanding

**C1** Refer to the Investigate on pages 426 and 427. The patio was divided into four simple shapes: a rectangle, two triangles, and a semicircle. Describe how to determine the area of the patio by adding the areas of two shapes.

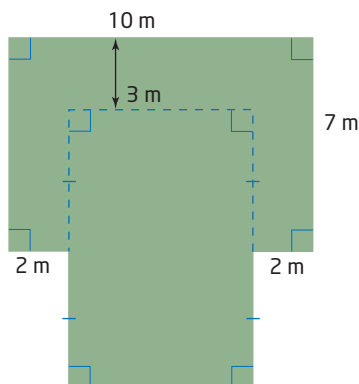
**C2** Consider the yard shown.

- Describe how you can determine the unknown lengths.
- Describe how you can determine the area of the yard by adding the areas of simpler shapes.
- Describe how you can determine the area by subtracting areas.



- C3**
- Suppose you need to calculate the perimeter of the yard in question C2. Explain why you cannot simply add the perimeters of the rectangles that make up the composite figure.
  - Without calculating, describe how the perimeter of this yard compares to the perimeter of a rectangular yard that measures 10 m by 7 m.

- C4**
- How does the perimeter of the yard in question C2 compare to the perimeter of the yard shown, which has been increased by the smaller rectangular section rather than being decreased in size?
  - Describe how you would determine the area of this yard.
  - How does its area compare to the area of the yard in question C2?

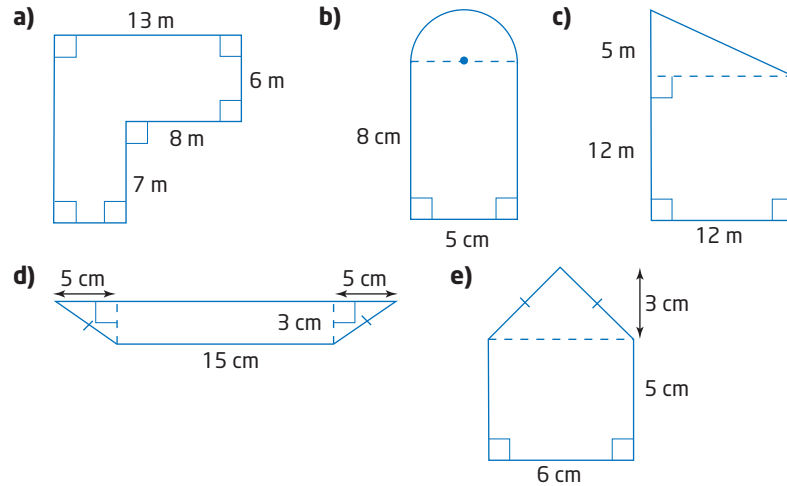


## ■ **Practise**

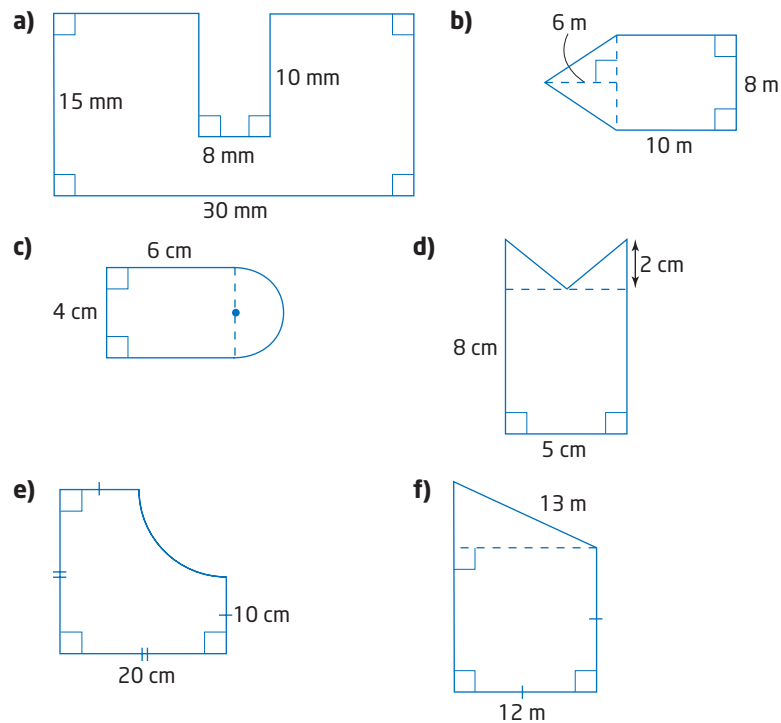
For help with questions 1 and 2, see Examples 1 and 2.

1. For each composite figure,
  - solve for any unknown lengths
  - determine the perimeter

Round to the nearest unit, if necessary.

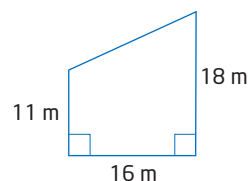


2. Calculate the area of each composite figure.  
Round to the nearest square unit, if necessary.

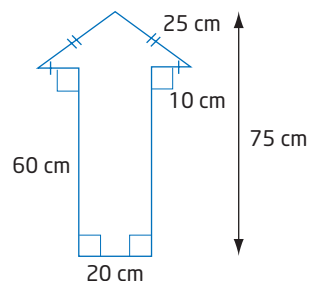


## Connect and Apply

3. a) What length of fencing is needed to surround this yard, to the nearest metre?  
 b) What is the area of the yard?  
 c) Explain the steps you took to solve this problem.



4. Patrick is planning a garage sale. He is painting six arrow signs to direct people to his sale.
  - a) Calculate the area of one side of one arrow.
  - b) Each can of paint can cover  $2 \text{ m}^2$ . How many cans of paint will Patrick need to paint all six signs?
  - c) If the paint costs \$3.95 per can, plus 8% PST and 7% GST, how much will it cost Patrick to paint the six signs?



5. Arif has designed a logo of her initial as shown. Use a ruler to make the appropriate measurements and calculate the area of the initial, to the nearest hundred square millimetres.



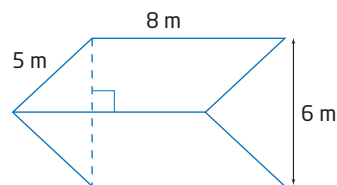
6. Create your own initial logo similar to the one in question 5. Calculate the total area of your logo.

## 7. Use Technology

- a) Use *The Geometer's Sketchpad*® to draw your design from question 6.
- b) Use the measurement feature of *The Geometer's Sketchpad*® to measure the area of your design.

8. **Chapter Problem** One of the gardens Emily is designing is made up of two congruent parallelograms.

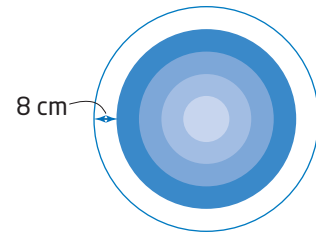
- a) A plant is to be placed every 20 cm around the perimeter of the garden. Determine the number of plants Emily needs.
- b) Calculate the area of her garden.



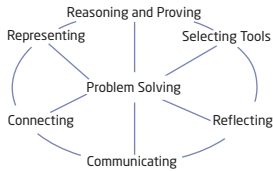
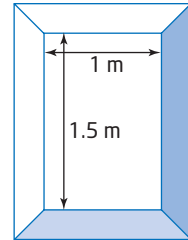
9. **Use Technology** Use *The Geometer's Sketchpad*® to create a composite figure made up of at least three different shapes.
  - a) Estimate the perimeter and area of the figure you created.
  - b) Determine the area using the measurement feature of *The Geometer's Sketchpad*®. Was your estimate reasonable?



10. An archery target has a diameter of 80 cm. It contains a circle in the centre with a radius of 8 cm and four additional concentric rings each 8 cm wide.

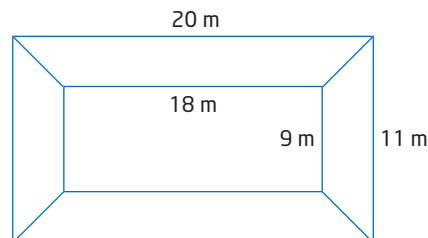


- a) Find the area of the outer ring, to the nearest square centimetre.
  - b) What percent of the total area is the outer ring?
11. The area of a square patio is  $5 \text{ m}^2$ .
- a) Find the length of one of its sides, to the nearest tenth of a metre.
  - b) Find the perimeter of the patio, to the nearest metre.
12. Brandon works as a carpenter. He is framing a rectangular window that measures 1.5 m by 1 m. The frame is 10 cm wide and is made up of four trapezoids. Find the total area of the frame, to the nearest square centimetre.



### Achievement Check

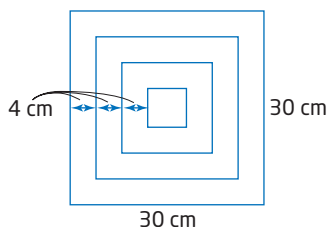
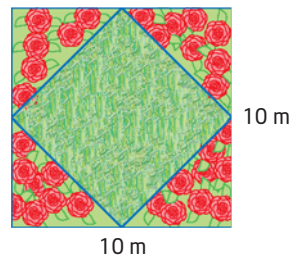
13. Susan is replacing the shingles on her roof. The roof is made up of a horizontal rectangle on top and steeply sloping trapezoids on each side. Each trapezoid has a (slant) height of 4.5 m. The dimensions of the roof are shown in the top view.



- a) Calculate the area of the roof.
- b) A package of shingles covers  $10 \text{ m}^2$ . How many packages will Susan need to shingle the entire roof?
- c) Describe an appropriate way to round the number of packages in part b).

## Extend

14. Sanjay is designing a square lawn to fit inside a square yard with side length 10 m so that there is a triangular flower bed at each corner.
- Find the area of Sanjay's lawn.
  - How does the area of the lawn compare to the area of the flower beds?
  - Sanjay's design is an example of a square *inscribed* within a square. The vertices of the inside square touch the sides of the outside square but do not intersect. Will your answer in part b) always be true when a square is inscribed within a square? Explain.
15. How does doubling the radius of a circle affect its area? Justify your answer using algebra.
16. Leonardo of Pisa lived in the 13th century in Pisa, Italy. He was given the nickname Fibonacci because his father's name was Bonacci. Among his mathematical explorations is the sequence of numbers 1, 1, 2, 3, 5, 8, 13, 21, ....
- Determine the pattern rule for this sequence, and list the next four terms.
  - Construct rectangles using consecutive terms for the sides. The first rectangle is 1 by 1, the second is 1 by 2, the third is 2 by 3, and so on. Find the area of each rectangle.
  - Explore the ratios of the sides of the rectangles. Make conjectures about this ratio.
  - Explore the ratios of the areas of the rectangles. Make conjectures about this ratio.
17. **Math Contest** Determine the ratio of the perimeter of the smallest square to the perimeter of the largest square.



18. **Math Contest** The midpoints of the sides of a rectangle that measures 10 cm by 8 cm are joined. Determine the area of the shaded region.

